Trigonometry

Trigonometric Formulas

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$1 + \tan^2 A = \sec^2 A$$
$$\cot^2 A + 1 = \operatorname{cosec}^2 A$$

Addition Formulae

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$
$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$
$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$
$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$
$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$
$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Double Angle Formulae

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

R-Formula (**NOT** provided in O-Level formula list)

$$a \sin \theta \pm b \cos \theta = R \sin(\theta \pm \alpha)$$
$$a \cos \theta \pm b \sin \theta = R \cos(\theta \mp \alpha),$$

Where $R = \sqrt{a^2 + b^2}$ and $\tan \alpha = \frac{b}{a}$

Additional Trigonometric Identities

Negative Angles of Trigonometric Identities:

$$\sin(-x) = -\sin x \quad \cos(-x) = \cos x \quad \tan(-x) = -\tan x$$

Special Acute Angle Transformations

$$\sin(90 - \theta) = \cos \theta \quad \cos(90 - \theta) = \sin \theta \quad \tan(90 - \theta) = \frac{1}{\tan \theta}$$

Further Trigonometric Identities:

$$\operatorname{cosec} x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

Radian Measure

$$\pi \, rad = 180°$$

Converting radians to degrees: $1 \, rad = \frac{180}{\pi}$

Converting degrees to radians: $1° = \frac{\pi}{180}$

Special trigonometric values

x	0°	30°	45°	60°	90°
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan x$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undefined

Trigonometric Graphs

Graphs of $y = a \sin bx$, $y = a \cos bx$ and $y = a \tan bx$
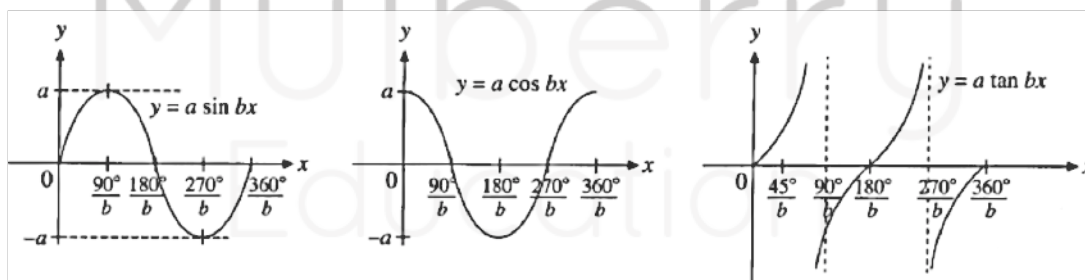
Amplitude $= a$
Period $= \frac{360°}{b}$

Amplitude $= a$
Period $= \frac{360°}{b}$

No amplitude
Period $= \frac{180°}{b}$

Different Types of Trigo Graphs:

(a) Changing the amplitude $eg. \, y = a\sin x$

(b) Changing the period $eg. \, y = \sin ax$

(c) Translation of the graph $eg. \, y = \sin x + a$

(d) Negative function (reflection about the $x-$axis) $eg. \, y = -\sin x$

(e) Modulus Functions (Graph stays above the $x-$axis) $eg. \, y = |\sin x|$

(f) Or any combination of the above